The Practical Guide To Physical Units for Particle Measurements

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Introduction:

The following summary of physical quantities for particle measurements and their dimensions is intended for reference. The approach taken proceeds from the raw measurements toward the phase space density concept. The term "effective width", as used herein, refers to the width of the rectangular representation of a response function having the same area as the actual response function, and the same height as the response function at its nominal centroid (i.e., the effective area).

Count Rate:

The particle count rate is given by:

$$R_C[Hz] = \frac{N[counts]}{s[samples]\tau[\sec/\,sample]} \tag{1}$$

where N is the number of particles in a specific accumulation, and s is the number of basic accumulation samples, each of duration τ, that contributed to the total accumulation.

Number Flux:

The particle differential, directional number flux, I (also referred to as intensity, for example, in radiative transfer theory), corresponding to a specific counting rate of particles is given by:

$$I[s^{-1}cm^{-2}sr^{-1}eV^{-1}] = \frac{R_C[Hz]}{dA[cm^2] \bullet dAz[rad] \bullet dL[rad] \bullet dE[eV]} \tag{2}$$

where dA is the effective area that the sensor presents to the exterior environment, dAz is the effective width of the angular aperture in one dimension, dL is the effective width of the angular aperture in the orthogonal dimension, and dE is the effective width of the energy aperture (passband). The product of dA, dAz, and dL is referred to as the geometric factor, with dimensions of [cm^2 sr]. Thorough calibration work will yield values of this integral factor as well as measurements of the individual response widths and effective area. For electrostatic energy analyzers, dE is proportional to the centroid energy E_0, so dE = (dE/E)•E_0. The factor (dE/E) is sometimes included in the geometric factor even though it is dimensionless. For an integral instrument like DE/RIMS, the factor dE is ommitted and only the integral number flux can be specified.

Energy Flux:

The particle differential, directional energy flux corresponding to a specific counting rate of particles is given by:

$$\mathcal{E}[eV\ s^{-1}cm^{-2}sr^{-1}eV^{-1}] = \frac{R_C[Hz] \bullet E_0[eV]}{dA[cm^2] \bullet dAz[rad] \bullet dL[rad] \bullet dE[eV]} \tag{3}$$

Note that for electrostatic energy / charge analyzers, the factor E_0 may be cancelled:

$$\mathcal{E}[eV\ s^{-1}cm^{-2}sr^{-1}eV^{-1}] = \frac{R_C[Hz]}{dA[cm^2] \bullet dAz[rad] \bullet dL[rad] \bullet (dE\ /\ E)} \tag{4}$$

The simplicity of the relationship between count rate and energy flux explains the attraction of referring to the denominator here as the geometric factor.

Phase Space Density:

The phase space density may usefully be thought of as the product of the more familiar configuration space density, n [cm^{-3}], and the velocity space distribution, f [$cm^{-3}s^3$]. It is given by the following expression:

$$f[cm^{-6}s^3] = \frac{dN}{d^3x[cm^3]\, d^3v[cm^3s^{-3}]} \tag{5}$$

Using the relationship between energy and velocity [$dE = m\ v\ dv$, therefore $d^3x\ d^3v = dx\ d^2x\ v^2 dv d\Omega = (dx / v)\ dA\ v^2\ (dE / m)\ d\Omega = dt\ dA\ d\Omega\ dE\ (v^2 / m)$], it can be shown that the phase space density is related to the differential directional flux by a simple proportionality:

$$f[cm^{-6}s^3] = I[s^{-1}cm^{-2}sr^{-1}erg^{-1}] \bullet \left(\frac{m[g]}{v_0^2[cm\ s^{-1}]} \right) \tag{6a}$$

or

$$f[cm^{-6}s^3] = I[s^{-1}cm^{-2}sr^{-1}erg^{-1}] \bullet \left(\frac{m[g]^2}{2E_0[erg]} \right) \tag{6b}$$

where m is the particle mass, E_0 is the particle kinetic energy, and v_0 is the particle speed. When consistent units are used throughout (e.g., cgs, ergs, 1.602e-12 [erg eV^{-1}]), the dimensions are as shown. A more practical set of units for space applications is [$km^{-6}\ s^3$], for which the conversion factor is 10^{30}[$km^{-6}\ cm^6$]. When dealing with multiply charged ion species, it is important to use actual mass and energy values rather than energy / charge or mass / charge values that are more directly measured by most instruments ($E = Z \bullet E / Z$, $m = Z \bullet m / Z$, where Z is the charge state).